Workshop on "Age-structured models in population dynamics and economics" at the Vienna Institute of Demography, October 27–28, 2003

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The workshop brought together specialists in the theory and in different areas of application of age- and/or duration-structured dynamical systems. Among those areas are demography, population dynamics, economics, epidemiology, ecology and others. The main topics were:

- Dynamics and control of human populations
- Descriptive and control models in population economics
- Vintage models of capital accumulation and labour distribution
- Structured epidemiological models and control of infectious diseases
- Structured dynamic models in social sciences

All talks presented at the workshop had in common that age-structured populations (in a literal or more general sense) were involved. The workshop manifested the recent substantial progress in both the mathematical techniques employed in modelling of age-heterogeneous populations, and the scope of applicability of these techniques in different areas. The workshop corroborated the tendency to merge population and economic dynamic considerations on the basis of unified modelling. Another point to mention is the considerable presence of control models in a dynamic optimisation or viability context. Such models are employed not only from a normative point of view, but even more to understand the behaviour of presumably rational agents, or for the generation of non-trivial objective-oriented scenarios.

The talks presented at the workshop are briefly summarised below. The first group of talks address issues concerning purely biological aspects of populations, while the talks in the second group involve economic considerations.

Structured Populations in Demography and Epidemiology

In his talk *Endemic states of an immunising infection with positive case fatality in a growing population*, Klaus Dietz (Tuebingen/Germany) presented a joint paper with M. Safan and K. P. Hadeler on an extension for a growing population of the classical epidemiological model by Daniel Bernoulli for a potentially fatal infection in a constant population. Age-dependent birth and death rates are considered with a positive Malthusian parameter in the absence of the infection. Critical bounds for age-dependent case-fatality rates are derived which are compatible with an endemic state.

The force of infection is also allowed to depend on age. General formulas for the basic reproduction number in terms of the model parameters and in terms of the equilibrium prevalence of the susceptibles are derived. It is assumed that the duration of the infectious period is not affected by the mortality induced by the infection. The analysis is focused in particular on infections whose infectious period is of negligible duration compared to the life expectancy of an individual.

Rui Dilão (Lisbon/Portugal) discussed in his talk *Population cycles in the age dependent McKendrick model* the Easterlin cycles. These are oscillations in the population growth predicted by Easterlin in the 1960s with a very simple two-generation model. The author reported existence of demographic cycles in the context of a McKendrick model with age-dependent birth and death rates. For a population with only one reproductive age class, the temporal evolution of the population size is always modulated by a time periodic function with a period equal to the age of the reproductive age class. For a population with a continuous distribution of reproductive age classes, the existence of damped cycles is proved. The periodicity of the damped cycles is associated with the age of the first reproductive age class. Damping increases as the dispersion of the fertility function around the age class with maximal fertility increases. In general, the period of the demographic cycles is associated with the time that a species takes to reach reproductive maturity.

In his talk *Some results on the modelling of tuberculosis*, Mimmo Iannelli (Trento/Italy) presented a general epidemic model that includes some peculiar aspects of tuberculosis (TB). Long periods of latency and the emergence of antibiotic resistance due to incomplete treatment are very important features of TB dynamics. A two-strain TB model is formulated, with an arbitrarily distributed delay in the latent stage of individuals infected with the sensitive strain, and the effects of variable periods of latency on the disease dynamics are investigated. The author discussed the invasion and coexistence of the two strains versus a significant parameter such as the rate at which infected people develop drug-resistant TB, due to incomplete treatment. Control policies through screening programs are proposed for the case of a single (drug-sensitive) strain.

Hisashi Inaba (Tokyo/Japan) presented a talk on *Endemic threshold results in an age-structured population model for HIV/AIDS epidemic*. Within an age-duration-structured model of the HIV infection in a homogeneous community, the invasion problem of the HIV/AIDS epidemic is investigated using positive operator theory. Conditions for the existence and uniqueness of an endemic steady state are given. In particular, it is shown that a backward bifurcation can occur at the threshold, i.e., multiple endemic steady states can exist for the structured model of HIV infection. Stability results for the endemic steady states are discussed.

Nan Li (Berkeley/USA) presented the paper *Population momentum for real population and linear fertility transition.* Population momentum is the ratio of a population's ultimate size after a demographic transition to its initial size before the transition. For stable population and instantaneous drop to replacement fertility, Keyfitz found a simple formula for the momentum. Keyfitz's formula has been extended to

describe gradual demographic transitions. The stable initial population, however, still obstructs approaching reality. Extending the solution of a Lotka equation with time-varying vital rates to negative values of time, this paper provides an approximate formula for the momentum of any initial population that undergoes linear fertility transition. Examples using data from the United Nations indicate that this formula works well both for age structures far away from or close to stable.

Population Economics

Christian Almeder and Gernot Tragler (Vienna/Austria) presented a joint talk on *Optimal prevention programs in an age-structured drug initiation model.* A brief review of dynamic models of illicit drug use consisting of ODE's (both descriptive and normative) is given. Even though these models do a good job in approximating empirical data such as the numbers of 'light' and 'heavy' cocaine users in the U.S., they neglect one important fact, which is the strong dependence of drug initiation on age. Therefore a model for drug initiation that extends traditional dynamic models by considering explicitly the age distribution of the users is introduced. On the basis of a 2-groups model in which the population is split into a user and a non-user group the advantage of a continuous age distribution is shown by considering more details, yielding new results. Neglecting death rates reduces the model to a single state (1-group) descriptive model which can still simulate some of the complex behaviour of drug epidemics such as repeated cycles. Furthermore, prevention programs—especially school-based programs—can be targeted to certain age classes. Optimal control models are used for best allocation of resources to prevention programs over different age classes.

Noël Bonneuil (Paris/France) presented the paper *Preserving Transfer Benefit for Present and Future Generations* (jointly with R. Boarini). The main issue is: how should state interventions be calibrated in time and magnitude to guarantee a positive account for each generation as well as a minimal welfare and some equity across age classes? The social contract turns into the dynamic maintenance of a positive account for every present and future generation. It requires either accelerating or slowing down both pension and education spending. The ensuing dynamic trade-off between generational benefit and equity delineates an optimal interest rate in human capital. Moreover, the favourable effect of population growth on the capacity to pay for pensions is thwarted when fairness between age classes together with the preservation of a positive net benefit for future generations is also considered. The speaker presented an informal introduction to the viability theory, which is the main tool the authors employ in the analysis.

Raouf Boucekkine (Louvain/Belgium) presented the overview *Modelling vintage structures with differential-difference equations: specification, stability assessment and optimal control.* First a vintage capital model of the Solow type is discussed in order to simply illustrate the main specifications within this framework, and the

typical stability and numerical solution tools. Then Ramsey vintage capital models with and without a stationary environment are considered. The dynamic properties of such models, together with the associated analytical tools, are briefly summarised. Finally, two vintage capital models displaying endogenous growth are presented. The first one describes the dynamics of human capital accumulation under a realistic survival law and a vintage human capital structure. The second is the one-hoss shay model with constant return to capital. Handling this model the author provides a survey on some of the optimal control material required to treat the presence of lagged controls in this model.

Alexia Fuernkranz-Prskawetz (Vienna/Austria) presented the paper *Optimal age-specific labour force: effects of education, productivity and wages* (jointly with P. Kort and V. Veliov) which analyses the optimal hiring and firing policy of a single firm that is subject to hiring and firing costs. In contrast to previous contributions, this paper allows for age-dependent labour force, adjustment costs, wages, and productivity. In particular, the paper investigates the effect of the age-specific wage-productivity gap on the optimal age schedule of the work force.

Peter Kort (Tilburg/Netherlands) presented the paper *Environmental policy, the Porter hypothesis and the composition of capital: Effects of learning and technological progress* (jointly with G. Feichtinger, R. Hartl and V. Veliov). This paper investigates the effect of environmental policy on the composition of capital. By allowing for non-linearities it generalises an earlier paper by Xepapadeas and De Zeeuw (1999) and determines scenarios in which their results do not carry over. In particular, it is shown that the way acquisition cost of investment decreases with the age of the capital stock is of crucial importance. Also it is obtained that environmental policy has opposite effects on the average age of the capital stock in the case of either deterioration or depreciation. Furthermore, it turns out that in the presence of learning, implementing a stricter environmental policy with the aim to reach a certain target of emissions reduction has a stronger negative effect on industry profits, which implies quite the opposite from what the Porter hypothesis describes.

The paper *Optimal harvesting of age class structured forest resources* by Olli Tahvonen (Helsinki/Finland) studies optimal harvesting of multiple age class forest resources under discounting and strictly concave optimisation criteria. The problem is analysed as a discrete time, infinite horizon and any number of state and control variables optimisation problem. The necessary and sufficient optimality conditions are obtained using the Karush-Kuhn-Tucker theorem in non-linear programming. It is shown that at the optimal stationary state, timber harvest may evolve cyclically over time. The cycles occur because in the forest age class model space is measured continuously and time in discrete units. Cycles disappear as period length moves toward zero, when the optimal rotation period is not unique or when the discount factor approaches unity.

In his talk *Optimal taxation and governmental investment: an age-structured model*, T. Tsachev (Sofia/Bulgaria) presented a macroeconomic model in which the population and the capital are age-structured. The government's revenue consists of

a single tax, modelled as a percentage of the GNP. The model is in the form of an optimal control problem. The control variables are the collected tax and the percentage of it which the government spends for investments. The discounted social welfare is to be maximised over a prescribed time interval. Identification and solution approaches are discussed.

In the talk entitled *Heterogeneous control systems: a general perspective and applications in population dynamics and economics*, Vladimir Veliov (Vienna/Austria) presented the concept of a heterogeneous control system which extends that of age-structured systems by allowing for parameters of heterogeneity other than age or duration. The basic theory is briefly outlined, including necessary optimality conditions, second order sufficient conditions, and numerical approaches. Then some particular heterogeneous control models arising in demography, economics, and epidemiology are discussed, with more details devoted to a model of the spread of an infectious disease in a population which is heterogeneous with respect to the level of risky behaviour.